The Minister of the Admiralty has promised to send every instrument which should appear suitable for the purpose. The Committee of the German Chemical Society have elected superintendents to procure a worthy representation of chemical apparatus and specimens of chemical compounds of scientific interest. It has been decided to address an invitation to the members of the society to send such specimens to Berlin, in order to form a systematical and uniform collection of rare and interesting chemicals.

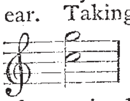
We may state generally that the Governments of Belgium, France, Germany, Italy, the Netherlands, and Switzerland, have now appointed committees to act in concert with the general committee in London. The Government of the United States has, through Mr. Fish, intimated that it is in communication with the various Departments and Scientific Institutions, with the object of forwarding the Exhibition.

BEATS IN MUSIC *

II.

THE second kind of beat differs from the first in that it arises from the imperfection, not of unisons, but of wide-apart consonances, such as the third, fourth, fifth, sixth, and octave.

This beat is well known practically to organ tuners, and may be soon rendered appreciable to any musical ear. Taking the fifth as an example, let the two notes



be sounded on an organ, or any instrument of sustained tones. If they are perfectly in tune, the united sound will be smooth and even; but if one of them be sharpened or flattened a little, a beat will be heard just as in the case of the imperfect unison; and which, like it, will increase in rapidity as the note is made more and more out of tune. That this is not the same beat as Tartini's is obvious from the failure of the rule for the latter when applied to the former; for example, when the concord is in tune the upper note vibrates 768, and the lower one 512 vibrations per second, therefore there ought to be 256 beats per second; but in reality there are no beats at all, they only begin when the notes are put out of tune; hence the Tartini-beat rule is useless and inapplicable in this case.

This beat may be called the *consonance* beat, and it has also been termed "Smith's beat," from its having been first investigated by him.

The theory of Smith's beat, as given by Smith himself, is complicated and difficult to describe; but we will endeavour to give some idea of its nature and cause.

We must return to the illustration of the coffin-makers. Suppose one of them to have sold his business to another man in the trade, who was so much more active and energetic that he could drive his nails half as fast again as ordinary workmen. Call him A, and suppose that when he began to work it was found that he struck exactly three blows to two of his neighbour B. As B struck 100 per minute, A will now strike 150. And assume that on a certain day they both begin exactly together. The passer-by will hear that every third blow of A exactly coincides with every second of B; so that he will notice fifty coincidences in a minute; or to describe them more correctly, he will notice per minute fifty *phases of compound effects*, in each of which there is a coincidence. This phase constitutes *Tartini's beat*, but now very much augmented in rapidity from what it was before: then there were only one or two coincidences per minute, now there are fifty.

Now suppose that A, from some slight exhilarating cause, begins to strike a little faster; *i.e.* that he makes 151 blows in a minute instead of 150. Let us endeavour to find out what will be the result on the listener. Still supposing the two strokes to begin with a coincidence, the third blow of A will still coincide *very nearly indeed* with the second of B; it will only differ from it by $\frac{1}{7500}$ of a minute, a quantity inappreciable to the ear. Hence the Tartini phase will at this time be practically unharmed. But after a few repetitions the divergence of the blows will be so great as to become appreciable, and the listener will begin to notice a series of *changes of form* of the Tartini phase, in which there is now *no coincidence* of the blows, but only a variation of their arrangement, which, moreover, is itself constantly varying. After a time, however, these changes will exhaust their possible varieties, the listener will notice that two of the blows begin to approach again, and at last will *coincide*, as they did before. He thus notices a *long cycle* of the Tartini beats, and this long cycle is the *Smith's beat*. It is, in fact, a beat of what mathematicians would call the second order; the first, or Tartini's beat, is a cycle of differing periods; the second, or Smith's beat, is a cycle of differing cycles.

Let us next attempt a numerical estimation of the length of this second cycle in the case of the coffin-makers. To effect this we must inquire when the coincidences of two blows will recur. It is plain that they will recur at the *end of the minute*, i.e. if the first blow of A coincided with the first of B, then the 151st of A will coincide with the 100th of B. This will give one long cycle, or one Smith's beat, per minute. A careful comparison of the times of the respective blows will show, moreover, that (since 100 and 151 are prime to each other) there will be *no other* exact coincidence during the minute; and a hasty reasoner may conclude that one beat per minute will be the proper number. But if the listener be asked to describe what he hears, he will dissent from this and say confidently that there are *two* places in the minute where he hears a coincidence. To test his assertion, let us apply Young's principle mentioned before, and inquire whether in the course of the minute there is any other place where the blows *so nearly* coincide that the ear may mistake them for real coincidences. The 74th blow of A will occur at $\frac{74}{151}$ of a minute after starting, whereas the 49th blow of B will occur at $\frac{49}{100}$ of a minute. The difference between these is only $\frac{1}{15100}$ of a minute, which is quite inappreciable. Hence, practically, there will be two parts of the minute where the blows coincide, and there will be consequently two Smith's beats in the same time.* If we were to suppose A to make 152 blows per minute (or 148, for a deficiency would produce the same result as an excess) to B's 100, we should, calculating on the above plan, find *four* cycles or beats per minute. Or we may alter the proportion: suppose for example, A, intending to make five blows to B's four, makes really 126 per minute instead of 125 as he ought to do; it will be found by calculation that there will be *four* places of coincidence in the minute, or four Smith's beats; if he strikes 127 blows, there will be eight Smith's beats—and so on.

We hope the foregoing homely illustration will help to render clear the nature of the Smith's beat as applied to sounds. Although the Tartini beat may not be really converted, as Young supposed, into the Tartini harmonic, but, according to Helmholtz, remains as a beat, inappreciable by reason of its great rapidity, it certainly has a physical and mathematical existence; and it as certainly changes its phases by reason of the small divergence of the times of vibration from those due to the true concord, and it is the recurrence of similar phases in a long cycle

* By W. Pole, F.R.S., Mus. Doc. Oxon. Continued from p. 214.

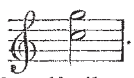
* Mr. De Morgan, in his admirable paper elucidating Smith's profound investigation, unfortunately omits to notice this important element of the *approximate* coincidences. The consequence is that his explanation is not easy to follow, and indeed would *appear* wrong, although his results are perfectly correct.

which gives rise to the phenomenon in question. Smith contrived, with profound ability, to account for and calculate the beat independently of the Tartini beat, or whatever it may be called, but the introduction of this by De Morgan has wonderfully simplified the comprehension of the thing.

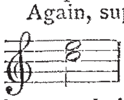
The accurate rule for finding how many beats per second will result from the concord being any given quantity out of tune; or for finding how much out of tune any concord is when it makes a certain number of beats per second, is remarkably simple.

Let n represent the denominator of the fraction, expressing, in the lowest terms, the true ratio of the concord (*e.g.* for the fifth $\frac{3}{2}$, $n = 2$; for the minor sixth $\frac{8}{5}$, $n = 5$, and so on); then let $q =$ the number of vibrations per second either in excess or deficiency of the number which would make the interval perfectly in tune; also let $\beta =$ the number of Smith's beats per second:

$$\text{then } \beta = nq,$$
$$\text{or } q = \frac{\beta}{n}.$$

A few examples will show the easy application of these formulæ. Take the concord of the fifth, .

When this is true the upper note should make 768 vibrations per second to 512 of the lower one; but if it is tuned by equal temperament the upper note will be slightly flat, making 767·15. Hence $q = 0{\cdot}85$; and as $n = 2$, we shall get $\beta = 1{\cdot}7$, *i.e.* there will be 102 Smith's beats per minute.

Again, suppose we find the concord of the major third  give 120 beats per minute ($= 2$ per second), how much is it out of tune? As in this case $n = 4$, we have

$$q = \tfrac{2}{4} = \tfrac{1}{2};$$

i.e. the upper note vibrates half a vibration per second either more or less than it ought to do.

The number of beats per second due to imperfections in the various consonances will be as follows, q being always the number of vibrations by which the upper note is untrue:—

Tartini's Beat.

For the unison	$\beta = q$.

Smith's Beats.

For the unison or octave	$\beta = q$.
" " fifth...	$\beta = 2q$.
" " fourth	$\beta = 3q$.
" " major third	$\beta = 4q$.
" " minor third	$\beta = 5q$.
" " major sixth	$\beta = 3q$.
" " minor sixth	$\beta = 5q$.

In the case of the *unison*, the Tartini beat and the Smith beat are synonymous, and this identity is the reason why so many writers on beats have gone wrong; they have so often taken unison sounds as the easiest and simplest for popular illustration, and have either assumed, without further investigation, that the same principles would apply for other consonances also, or have omitted notice of the other consonances altogether.

It will now be easy to understand why beats are capable of such great utility in a practical point of view—namely, as giving a means of measuring, with great ease and positive certainty, the most delicate shades of adjustment in the tuning of concordant intervals. To get, for example, an octave, a fifth, or a third perfectly in tune, the tuner has only to watch when the beats vanish, which he can observe with the greatest ease, and which will give him far more accuracy than he could possibly get by the ear alone. Whereas if he desires to adopt any fixed temperament, he has only to calculate the velocity of beats corresponding to the minute error which should be given to each concord, and the required note may be tuned to its proper pitch with a precision and facility which would be impossible by the unaided ear.

The delicacy of this method of tuning would hardly be believed, if it did not rest on proof beyond question. To recur to our example, the difference between 95 and 100 beats per minute would be appreciable by anyone with a seconds watch in his hand; and yet this would correspond to a difference of only $\frac{1}{24}$ of a vibration per second, or in pitch less than $\frac{1}{1000}$ of a semitone!

This use of beats has been long practised by organ-tuners to some extent, but its capabilities, as amplified by the aid of calculation, are certainly not appreciated or used as they ought to be.

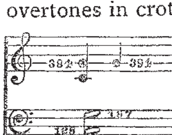
The third kind of beat is what we may call the *overtone* beat, and was brought into prominent notice in 1862 by Helmholtz, who uses it for important purposes in regard to his musical theories.

It is known that nearly all musical sounds are compound; they consist of a fundamental note, which is usually the strongest (and by which the pitch of the note is identified), but which is accompanied with several fainter and higher *harmonic* notes, or, as Helmholtz calls them, *overtones.* The first of these is an octave above the fundamental, the second a twelfth above, the third a fifteenth, the fourth and fifth seventeenth and nineteenth respectively, and there are others still higher which we need not mention here. The number and strength of the overtones vary for different kinds of sounds, but the five lowest ones are very commonly present and distinguishable. Now, suppose we sound two notes, having such a relation to each other that any of the overtones of one will come within beating distance either of the other fundamental, or of any of its overtones, then a beat will be set up, which is the kind of beat now in question.

A few examples will make this clear. The bass fundamental C shown by a minim in the following example, has its overtone 

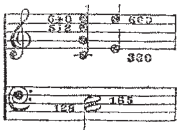
an octave above, as shown by the crotchet. Now if another fundamental C be sounded an octave above the former one, as the second minim, and if it be a little out of tune, there will be a unison beat between it and the overtone of the first note. This is one of Helmholtz's beats, and the simplest of them.

Again, take an interval of a fifth; the fundamental notes being shown in minims in the following illustration, and their respective overtones in crotchets:—



Here, if the G is not a perfect concord with the C, the two G's in the treble stave will be also out of tune with each other, and a unison beat will ensue. This is another Helmholtz beat, and a little more complex than the last, as both the beating notes are overtones.

Again, take an interval of a major third, expressing the notes and such of their respective overtones as we require in the same way as before, thus:—



Here, if we suppose the fundamental E to vibrate 165

instead of 160, *i.e.* five vibrations too sharp, the two upper E's in the treble stave will clash, and a beat will result.

In all these three cases Smith's beats also will naturally be present, and it is curious that in each case when we come to determine the rapidity of the beats, we find it come out the same, whether we calculate it by Smith's formula or by the unison beats of Helmholtz's overtones. We have added the vibration-numbers to the notes, to facilitate the calculation, and we find the number of beats per second to be—

For the imperfect octave ... ... = 5
For the imperfect fifth ... ... = 10
For the imperfect major third ... ... = 20

each arising from a sharpness of five vibrations in the upper note of the concord.

Hence we may lay it down as a principle that in consonances slightly out of tune, the beat given by the two fundamentals on Smith's plan, and those given by the first corresponding overtones on Helmholtz's principle, are synchronous, and may be considered identical.

The two kinds of beats, however, must not be confounded, as their cause is so distinct. The Helmholtz beats arise from the overtones only, whereas Smith's explanation applies to the fundamental notes, independently of the overtones altogether.

Helmholtz notices (Ellis's translation, pages 302-3) that beats of consonances will occur when sounded by simple tones, but accounts for them in another and very ingenious way, namely, by calling in the aid of the *grave harmonics*, or, as he calls them, the *combination tones.*

Taking our first example of the octave consonance given above, when the two notes of 128 and 261 vibrations are sounded together, they will give rise to a combination tone of 123 vibrations, and this, clashing with the 128 note, will give beats at the rate of five per second.

For the next example, the consonance of the fifth, this explanation will not suffice, and Helmholtz has to resort to a cause of the second order, namely, the beat of a grave harmonic, not with an imperfect unison, but with an imperfect octave. Taking our former example, an out-of-tune fifth C and G, of 128 and 197 vibrations respectively; these two notes will give a combination or difference tone of 69 vibrations, or an octave below the C, but out of tune. Then Helmholtz says this lower C will beat with its imperfect octave, on account of a new or *second order of difference-tones* formed from them, as in the former case.

In a similar but still more remote way, Helmholtz accounts for the beats of other consonances, the fourth, third, &c.

Without questioning the sufficiency of these explanations, I must say they seem to me somewhat far-fetched, and less satisfactory than Smith's, which account for the beats by a more positive and direct method, without calling in the aid of any sounds but the simple fundamental ones. There is at any rate the satisfaction that whichever explanation be adopted, the numerical value of the number of beats per second comes out the same and agrees with the fact; so that in a practical point of view it is immaterial which explanation be adopted.

I have alluded above to one important practical use of beats, namely, in tuning; but there is another use of them, also very interesting, *i.e.*, that they furnish a means of ascertaining the positive number of vibrations per second corresponding to any musical note. This may be done either by the unison or by Smith's beat, and I will give both methods.

For the unison beat:—Take two notes in unison on an organ, a harmonium, or other instrument of sustained sounds, and put one of them a little out of tune, so as to produce beats when they are sounded together. Let V and v represent the vibration numbers of the upper and lower notes respectively. Then by means of a monochord it will be easy to determine the ratio $\frac{V}{v}$, which call m. Count the number of beats per second, which call β. Then, since $\beta = V - v$, we obtain the simple equation,

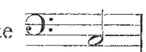
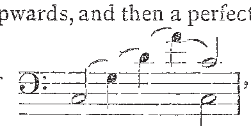
$$v = \frac{\beta}{m - 1}$$

which gives the actual number of vibrations per second of the lower note of the two.

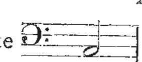
The method of deducing the vibration-number from the Smith's beat was pointed out by Smith himself; but as this method, so far as I know, is not to be found anywhere, except buried under the mass of ponderous learning contained in his work; I give it here in a simple algebraical form. If $\frac{m}{n}$ represents the true ratio of the interval, N the actual number of vibrations per second of the lower note, and M the same number for the upper one, the formula for Smith's beats becomes

$$\beta = \left(m - n\frac{M}{N}\right)N; \qquad \text{or } N = \frac{\beta}{m - n\frac{M}{N}}$$

Now, as m and n are both known for any given concord, if we can tell by any independent means the actual ratio of the notes $\frac{M}{N}$, we may, by simply counting the beats, calculate the actual number of vibrations N of the lower note. If the interval is too flat, β must be $+$; if too sharp, it must be $-$.

The following example will show how this may be done. Let it be required to determine how many vibrations per second are given by the note  on an organ. Tune three perfect fifths upwards, and then a perfect major sixth downwards, thus— , which will give the C an octave above the original note. But, by the laws of harmony, we know that this octave will not be in tune; the upper C will be too sharp, the ratio being $\frac{81}{40}$, instead of $\frac{2}{1}$, as it ought to be. Hence $\frac{M}{N} = \frac{81}{40}$, and $\frac{m}{n} = \frac{2}{1}$. Count the beats made by this imperfect octave, and suppose them = 192 per minute, or 3·2 per second; then, as the interval is sharp,

$$N = \frac{-3{\cdot}2}{2 - \frac{81}{40}} = 128\,;$$

i.e. the note  is making 128 double vibrations per second.

This method has the advantage of dispensing with the use of the monochord, which was necessary in the former case.

NOTES

A Meteorological Commission, appointed by the Ministers of Public Instruction, Agriculture and Commerce, Marine, and Public Works, to inquire into the possibility and practicability of a more intimate co-operation being effected among the various meteorological systems of Italy, have just issued their report. The Commission consisted of fourteen members, including most of the well-known meteorologists of Italy, with Prof. Cantoni as president, and Prof. Pittei as secretary, and met daily at Palermo from Aug. 30 to Sept. 6, 1875. The more important of the conclusions arrived at are these:—That all methods of observing at the stations of the various systems connected with the State be